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Industrial-Scale Designer-Driven Welding Optimization and Control Using an Integrated Computational Weld Mechanics Framework.

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Abstract: *A computational weld mechanics (CWM) framework that automates multiple setups, analyses and evaluations to explore a design space for design variations defined by Design of Experiment (DOE) matrices for a given design that is described. Saving an expert-user's time to prepare several analyses and allocating CPUs for efficient use makes this framework cost and time effective for managing industrial-scale designer-driven optimization and control applications of CWM. A validation analysis is conducted to identify the CWM control vector that minimizes the difference between the computed and experimental data. Actual CWM problems with continuous and discontinuous parametric design spaces including regression modelling surrogate modelling, sensitivity analysis, and control problems to minimize weld distortion are solved in this framework using derivative-free optimization algorithms that become attractive in this framework. The study demonstrates exploration of the design space for welding structures such as aircraft, ship, automotive and heavy machinery.*

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Introduction and Background

Computational welding mechanics (CWM) is a multi-physics problem that predicts the behaviour of welds in welded structures employing the models, algorithms, and software that are now mature and reliable. CWM is capable of supporting manufacturing design at the early stages of design to reduce the risk of high costs and long delays for correction and repair in manufacturing and service life. Unfortunately, there is a little application in routine engineering even in companies with welding core-competency.

Most academic CWM studies on complex industrial welding problems have focused on a simplified manual CWM implementation in combination with theoretical treatments that are very complicated and not feasible for industry to implement or use routinely. In industry the welding engineering tasks are limited by cost and delivery time. Heavy machinery industries, for example, need a weld sequence of several welding beads to be optimized in 30 days. This problem requires a combinatorial optimization with a significant number of analyses to be computed and compared which is challenging task to perform.

CWM models are complex for industrial users because the physics of welding is complex. This complexity means that the industrial user needs training and experience to setup and run a project. The expert-user time to prepare a project is significant and CPU time can be long. In addition, industrial CWM problems require solving several projects. In practice, a designer could not make a decision based on a single analysis and must evaluate many analyses. Human error in the setup of multiple analyses is very likely and managing several analyses may become too complex. In essence, a designer's skill is developed toward weld analysis and design space exploration but not to setting up complex computational analyses. The design team does not need to know how to setup the CWM project. The design team can focuses on design space exploration. This is similar to solving a math problem using Mathematica that enables the user to focuses on the problem independent of the mathematical complexity hidden behind the Mathematica commands.

It is misguided for the design team to spend time learning a complex computational skill. This is similar to expecting a Mathematica user to know the theory, details and setup, to evaluate the derivative of a complex function. In CWM, professional users of well-known software such as VrWeld, Sysweld, ABAQUS, or ANSYS strongly agree that meshing and preparing a welding project in actual industrial problems is very challenging and time-consuming. The authors believe that an expert in CWM setup can create an automated reference CWM design and ship it to the design team. They can implement the design of experiment (DOE) matrix to do designer-driven CWM optimization that could require hundreds of analyses with no detailed knowledge of CWM setup required by designers.

This has been the motivation of our work and the current paper discusses the support required to implement optimization in the design stage from a designer’s point of view and a framework with this capability. The framework uses DOE matrices as a communication language for a designer to explore a design space. The optimization methods to create DOE matrices that are well developed in statistics is not in the scope of this paper. If you have the analyses capability to solve a feasible problem, then the optimization and control algorithms to solve a sequence of problems is straight forward.

Computational Control and Optimization

Computational optimization of structures developed rapidly since 1970 but CWM remains a quite separate discipline. Few control and optimization papers have been published in the context of CWM. Michaleris [1] implemented an algorithm for a weld optimization problem using direct differentiation. This involves computing the derivatives of the governing equations with respect to the design parameters. For sufficiently smooth problems, optimization with direct differentiation is expected to be fast and accurate because it utilizes the gradient of the objective function. However, the disadvantage is that the code must be written to compute the gradient of the state equation or residual with respect to control parameters. Michaleris' direct differentiation was the main work for the derivative-based CWM optimization problem [1].

Derivative-free optimization works with the direct value of the objective function, and therefore the optimization requires the evaluation of an objective or cost function. A DOE matrix or sequence of DOE matrices to be evaluated is a convenient representation for such optimization. Voutchkov et al. [2] developed a surrogate model for a weld sequence optimization to minimize distortion in a tail bearing housing by analysing a DOE matrix of 27 tests chosen from the total space of 46,080 configurations. Tsai et al. [3] developed joint-rigidity-method (JRM) to determine the weld sequence to minimize the distortion in a thin-plate panel structure with 18 welds for Hyundai.

There are many effective optimization algorithms that could be employed in CWM. VanderPlatts [4] has a nice introduction to computational optimization for engineers. Nocedal, S. Wright [5] and Bertsekas [6] provide a more advanced mathematical viewpoint. However, lack of support for a robust and feasible implementation of their DOE matrices limits their practical use.

Optimization has three types; continuous optimization, combinatorial optimization and integer programming. In continuous optimization, the design variables are at least locally continuous functions. The optimization process starts with an initial guess or trial solution. The continuous optimization process then follows a path in the mathematical space defined by the design variables toward a min/maximum. The minimum requirement for continuous optimization is the capability to evaluate the cost function for any feasible set of design variables, i.e., at any point in the feasible design space. If in addition one can evaluate the gradient of the cost function with respect to the design variables if it exists, then the computing time can usually be reduced at the cost of implementing and validating the software support needed to evaluate the gradient. If in addition, the second derivative of the cost function can be evaluated if it exists, then computing time could usually be further reduced at the cost of more software development time. Using either or both the gradient and second derivative might make the setup more difficult and time consuming for the user. In combinatorial optimization, one seeks the optimal combination of some set of variables, e. g., choosing the sequence of weld joints or weld passes that minimize distortion is an important and challenging problem in welding. The fundamental mathematical structure in combinatorial optimization is a graph. The solution is the path in this graph that minimizes the objective function. A famous combinatorial optimization problem is the travelling salesman problem [7]. This class of problems is often very challenging. For example Tsai's [3] method needs n(n+1)/2 analyses, i.e., 171 for the Hyundai's thin-plate structure for a sequence of 18 DOE matrices with 18, 17, 16,..., 1 CWM analyses.

Computational Design of Experiment

A designer lives in a design/control space in which each axes is one design parameter. Every node in this space is a vector of one design configuration. There is another space which is the state or response manifold of the system. A map between each node in the design space to a node in the state space requires a CWM analysis. The state space is a vector space and hard to compare in the sense of picking the best. There exists a scalar objective function that is defined by the user as a selection criterion. Therefore, another map is required from the state space to the objective function which is usually a scalar function [8].

A designer decides which design configuration is to be evaluated in order to explore the design space. If it is written in the form of a matrix, each row is one CWM analysis and each column is the set of values for one design parameter. This is the representation for the computational design of experiment (DOE). Having a DOE matrix, the designer should not have to care how it is implemented. What is useful for the designer is to have the values of the objective function for each row of the designed DOE matrix to make decision. Therefore the DOE matrix is the communication language between a designer and a machine that automates the mapping between the spaces, and returns the values of the objective function for each row of the DOE matrix. Actual problems can use a DOE matrix or sequence of DOE matrices that require multiple analyses.

Integrated Optimization with CWM

Computational analysis on welding dates back to 1970 when Ueda [9] conducted the first computation of residual stress in welds. From 1980 to 2000, CWM research evolved rapidly and, then after 2000, CWM is being adopted quickly by industry. However, in the authors' judgment, CWM is currently not well integrated with optimization techniques. Since 2007, the authors have worked to integrate computational optimization and CWM. In this paper, a brief overview of several problems is provided to summarize what has been accomplished.

Verification and Validation

One of the essential tasks in CWM applications is the Verification and Validation of the computational code for a particular welding application. This requires an estimate of the difference between experimentally measured parameters and parameters computed by a computational model including the estimate of the uncertainty in both the experimental data and the computational data. If the difference in the results from valid experiments and predicted by a verified CWM model is sufficiently small, then the CWM model is validated for this particular welding application. In the authors' experience [10] the greatest source of error is in specifying the control vector, i.e., the parameters that characterize the weld experiment. Once errors in the control vector have been reduced sufficiently, then errors in material composition, temperature dependent material properties, and evolution of microstructure become important.

Masabuchi [11] published results of a careful experiment that measured thermal, strain and deflection on an edge-weld on a 152 x 1220 x 12.5 mm bar of Aluminum 5052-H32 using columns of four thermocouples, four strain gauges, and a dial gauge illustrated in Figure 1.

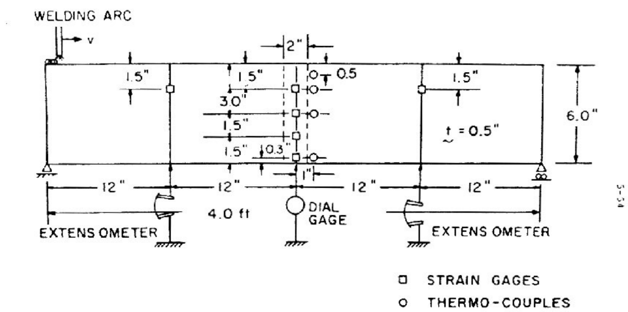

Figure 1 Masabuchi's experimental test setup.

Masabuchi's data was employed in our laboratory using VrWeld [12] to simulate his welding experiment. The temperature dependent material properties of Al 5052-H32 given in [11] were employed in the analysis. The gas metal-arc-welding process was employed to weld the specimen and the welding parameters were current 260 amperes, voltage 23 volts, travel speed 7.34 mm/s, filler metal Al-4043 with 1.6 mm wire diameter, wire feed speed 170 mm/s and the shielding gas was Argon. The specimen was allowed to cool to ambient temperature after welding was completed. The mesh shown in Figure 2 is finer close to weld path on the top edge with a total of 6600 8-node brick elements and 9680 nodes. The final distortion is shown in Figure 3 together with plots that compare Masabuchi's experimental data with computational results. The validation involves a sequence of DOE implementation for control vector calibration for get computed results as close as possible to the experimental data. This could also be viewed as computing the sensitivity of the experimental results to the control parameters. In other words, the objective function is minimizing the difference between an array of experimental and computational vectors.

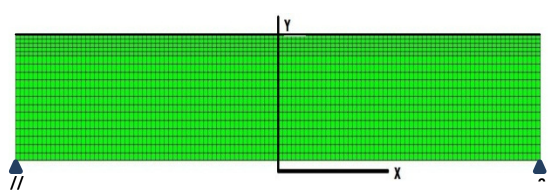

Figure 3 mesh employed in simulation of Masabuchi's test.

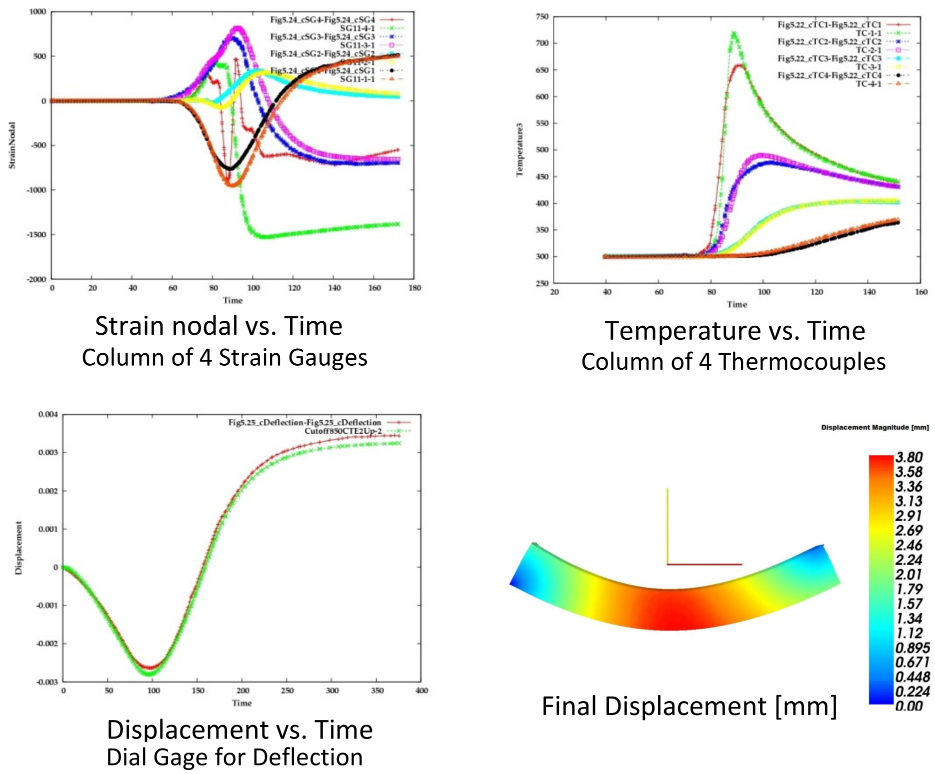

Figure 2 comparison between Masabuchi's experimental data with computational results.

Mitigation of Distortion by Clamping

The objective was to minimize distortion for the Masabuchi's edge welded bar with respect to clamping parameters using parametric design space exploration. Parametric design discretizes the space by given step sizes to create a grid on which each node is one design configuration. The design parameters in this problem were clamping parameters, i.e., prescribed displacements, and the release time after the weld halts. The design parameters; values of prescribed displacement and delay time, have a quite large range of possible variation and therefore the design space is discretized by picking 5 values of nodal prescribed displacement and 9 delay times resulting in 45 nodes in the discrete design space. A full factorial DOE including the 45 nodes was used to give a fully-covered map of the design parameters. The bar was then fixed at both ends and subjected to a range of prescribed displacements opposite to the direction of the camber. In the first set of tests, a prescribed displacement was directly applied in the middle of the bar. In the second set of tests, a parabolic displacement was prescribed along the full length of the bottom of the bar. Each set of tests had its own grid of 45

CWM analyses for a total of 90 CWM analyses. In addition, the effect of the delay times at which the prescribed displacement is released after the weld was studied.

The grid's nodes in the design space are defined by the DOE matrix, i.e., 2 DOE matrices of size 45 by 2. The DOE matrix was the main input file to the software. This is quite different from the user using the DOE matrix to separately create and separately solve one project for each row of the DOE matrix. Here, the user sets up only one reference or base project and two DOE matrices with total of 90 design points. The designs in each DOE matrix are run as a single project that analyses all 45 design points in 18.4 CPU hours on a single core of a 3.3 GHZ Intel quad-core processor. The user spends no time to set up the analysis for any design point other than the design point for the base or reference project. Figure 4 shows the final distortion after each clamping strategy which indicates significant mitigation comparing to the distortion in the reference design. See Figure 3. There was no single optimum for this problem and Figure 5 shows the curves of the best pairs for prescribed pre-bending and delay time that minimize the deflection. Details may find in [13].

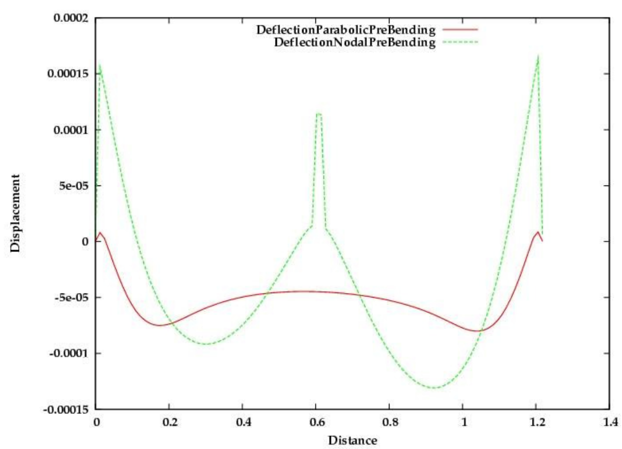

Figure 4 final distortions after each clamping strategy.

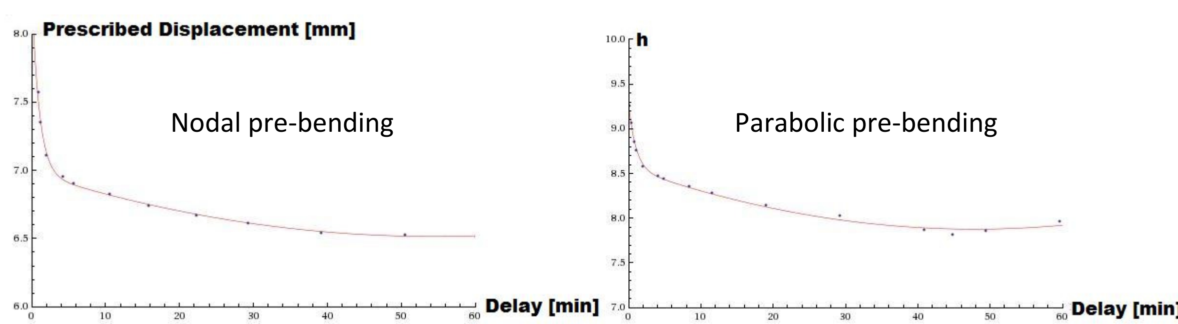

Figure 5 the best pair of prescribed displacement and delay time that minimize the deflection.

Mitigation of Distortion by Side Heating

A side heater mitigation technique was employed as another strategy for alleviation of Masabuchi`s bar. This technique applies a transient thermal tension by side heaters moving parallel to the weld path. The side heater's powers, heated area, the distance from the weld either longitudinal or transversal are the design parameters for this technique to be optimized.

In a continuous response surface of a given system, if the initial pattern does not find the optimum, direct-search algorithms can be used to learn from the current observation and find a possible path toward an optimum. The algorithm repeats the learning to follow the path until it reaches the minimum or is halted by some imposed limits.

We employed a recent direct-search algorithm from Kolda, Lewis and Torczon [14]. We also developed a modification to use a least-square approximation to improve the method of following a path to the minimum in the algorithm. This algorithm checks the value of objective function starting from an initial guess and surrounding variation then moves to the best value and repeats until it converges to the optimum point. Such algorithms requires performing a sequence of DOE matrices including tens of welding tests with different configuration of design parameters to follow the path to optimum design. The analysis consists of 12 sequences of 47 DOE matrices. The total CPU time was 12 hours and 48 minutes. The user time to set up the project was only to create the DOE matrices for the design. The base project from pre-bending problem was used and therefore there was no need for any effort to setup a new base project. Details may find in [15]. Figure 6 shows the final distortion for no-mitigation and side-heating-mitigation techniques.

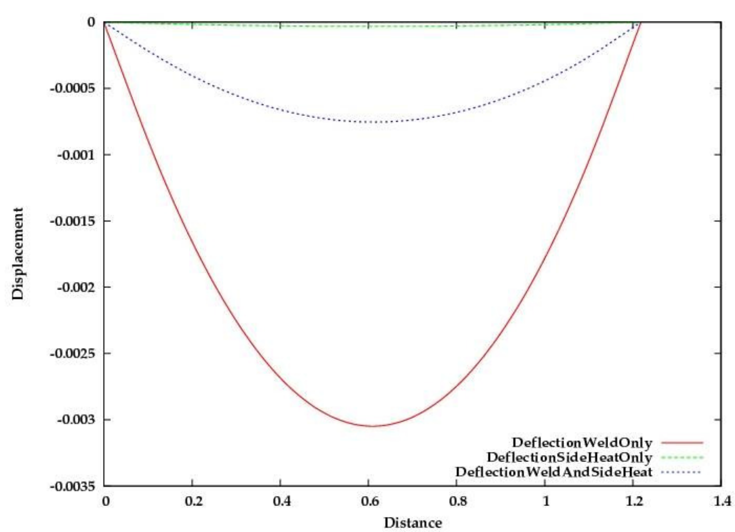

Figure 6 the final distortion for no-mitigation and side-heating-mitigation techniques on Masabuchi`s edge welded bar.

Residual Stress Variation due to Welding

The welding process generates residual stress in the structure before in-service loading and this residual stress changes the in-service behaviour of structure. The residual stresses are usually not known to design engineers in many cases and lack of such knowledge leads to conservative design practices that are expected to increase costs and reduce the performance of welded structures. It is expensive to experimentally measure residual stresses and therefore prediction of residual stresses by numerical modelling is a desirable complement to experimentally measured residual stresses.

Paradowska et al. [16] presented a set of experimental data of residual stress measured by neutron diffraction. The residual stress was measured at room temperature in the unrestrained specimen after the plate cooled down. Their experimental set up for the weld is shown in Figure 7. Paradowska`s work was simulated and effective stress is also shown in Figure 7.

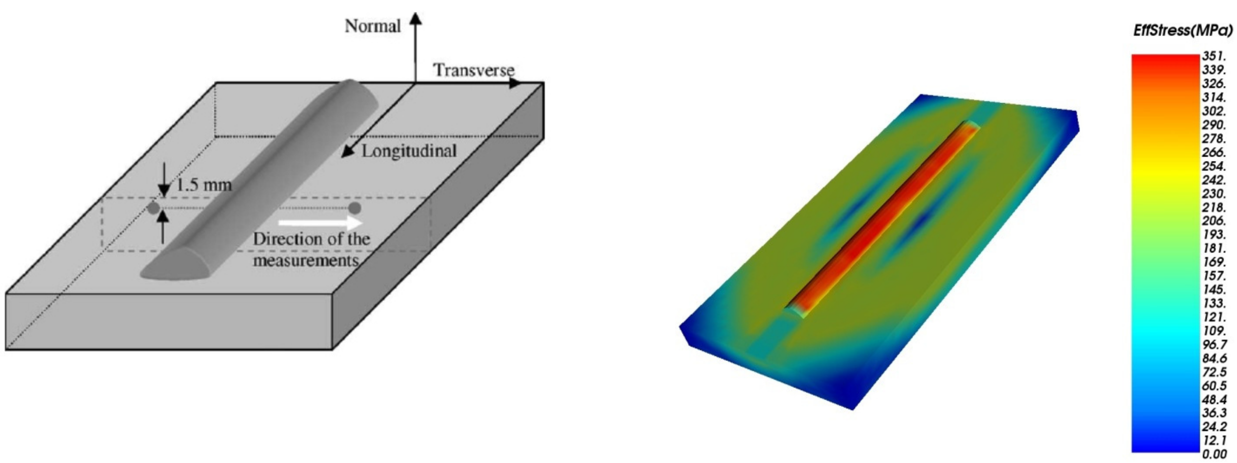

Figure 7 Masabuchi's experimental setup and simulation of his work.

The specimen is a low-carbon steel plate 100 x 200 x 12 mm. The filler metal is 14 mm wide and 6 mm high. The flux-cored arc welding process used a 1.6 mm diameter electrode with a 20 mm contact tip to work distance. ARGOSHIELD 52 shielding gas was used with a gas flow rate of 18 l/min. The welding parameters were 260–280 amps, 28–30 volts and a welding speed of 6 mm/s. During welding the specimen rested on a plate but was free to move with no constraints other than rigid body motion.

A reliability-based design was the objective of this problem that needs to know how the uncertainty in design parameters changes the results. Monte Carlo is a robust algorithm to use in such cases. Increasing the number of design parameters raises the number of points in the sample space and therefore the number of rows in a DOE matrix for the Monte Carlo analysis.

Initially the analysis deals with one parameter, i.e., welding current, and 30 non-repetitive samplings from a normal distribution around 280 amp with standard deviation 5 generates a DOE matrix of size 30 x 1 for different welding currents utilizing 4 cores. Using an accurate model, this DOE matrix gives the distribution of residual stress, i.e., objective function, with respect to the variation in welding current. The second part has four parameters of arc-weld-pool shape and a DOE matrix of 81 x 4 evaluations are used to construct a regression response surface as an estimator. This response surface is used for a Monte Carlo analysis with 10,000 random sample points from possible values of the parameters to observe the variation in the residual stress in the space of these parameters. Figure 8 left shows the variation of residual longitudinal stress along a line from the weld bead centreline to the side while arc weld current varying randomly and Figure 8 right shows the histogram of distribution 7.5 mm distance from the centre of the weld bead on the plate.

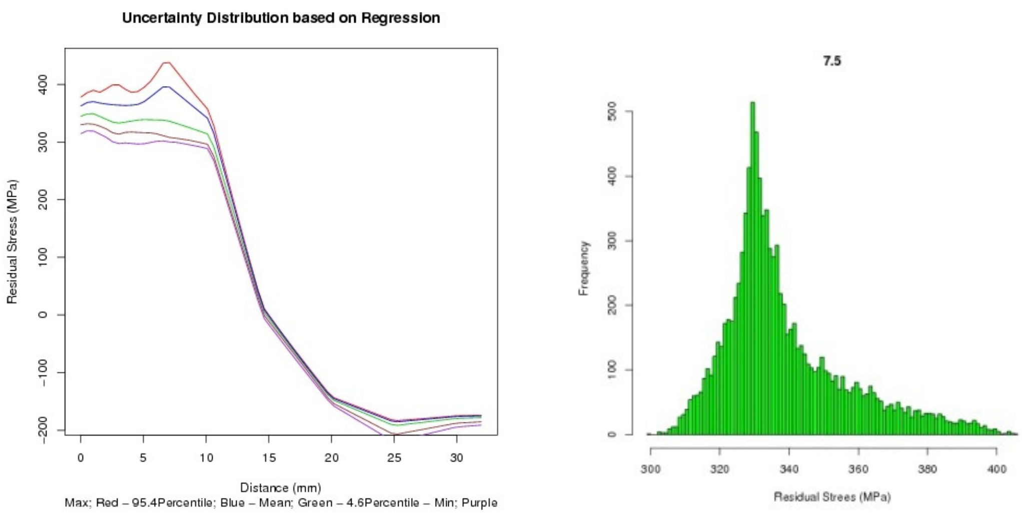

Figure 8 variation of residual longitudinal stress while arc weld varying randomly.

Hot Cracking and Ductility Dip Cracking in Weld

Computational weld mechanics (CWM) is used to estimate the likelihood of hot crack nucleation in a weld joint. A Solidification, Liquation or Ductility Dip hot crack nucleates when the evolution of the local state of stress, strain, temperature, and microstructure in the brittleness temperature range (BTR) reaches a critical value [17]. Solidification and Liquation Hot Cracks are associated with melting. In contrast, ductility dip cracking (DDC) occurs in a solid phase. The cause of the drop in material resistance to DDC nucleation at the ductility dip temperature range (DTR) has been attributed to an accumulation of voids, element segregation to grain boundaries, grain size, grain boundary orientation or a combination of these factors [18].

The local evolution state is determined by a high-resolution 3D transient CWM analysis and compared to experimental data characterizing the material resistance for each type of hot cracking. An algorithm determines the hot cracking risk based on the temperature, temperature profile, strain increment, and rate of strain in the hot cracking temperature region. The critical values are obtained from existing experimental data. The CWM model for the hot cracking test was based on the experimental procedure developed by Matsuda et al. [19] on an Inconel 600 plate 300x50x2 mm (Figure 9). The welding speed was 2 mm/s. This test relied on a cross-head speed (CHS) applied at the instant of time the heat source reaches the mid-point of the weld path, for a maximum duration of 3 s. Different CHS were used. The CWM model for the DDC test was based on the CWM simulation conducted by Chen and Lu [20] for a Filler Metal 82 (FM82) plate 100x100x2 mm using welding speeds of 2 and 5 mm/s. This problem demonstrates the capability of using post processors with DOE matrices. Details are in [21].

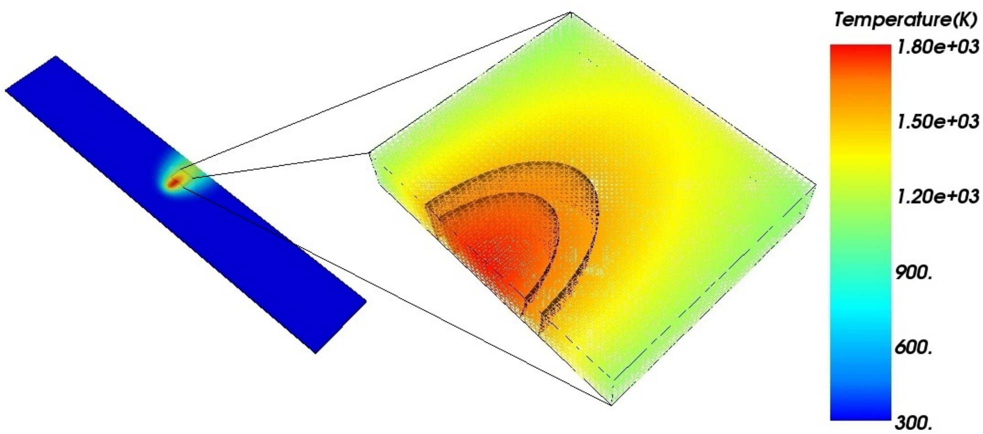

Figure 9 the brittleness temperature range (BTR) for Matsuda's test

Welding Sequence Pattern Optimization

Choosing an optimal sequence from the set of all possible combinations of a weld's sub-passes has been always a challenge for designers. The solution of such combinatorial optimization problems is limited by the available resources. Using a surrogate model based on a simulation model the solution in the space of all possible combinations can be found with a significant decrease in computational expenses. The discontinuous surrogate model constructs an approximation model from some combinations of sub-sequences of a more expensive model to mimic the behaviour of the expensive model as closely as possible but at a much lower computational cost. This surrogate model could be used to approximate the behaviour of the weld sequences not analysed with the expensive CWM model. We developed and demonstrated that a surrogate model can minimizes the distortion in a pipe girth weld with six sub-passes by analysing only a few combinations of sub-passes from total of possible combinations with the expensive model. This project was a pipe girth weld. Pipe girth welds are widely used in a variety of engineering applications such as oil and gas industries, nuclear and thermal power plants and chemical plants. The girth weld connects two pipes of length 356 [mm], wall thickness 17.5 [mm] and outer diameter 324 [mm]. The pipes are part of a longer pipe loop and therefore constraints are applied in the analysis to approximate the effect of the rest of the loop. The weld joint has 5 layers of weld. Each layer is divided into three sub-passes and each sub-pass covers 1/3 of the whole layer. The two pipes are tack-welded prior to welding at three points and each sub-pass starts from one tack-weld and ends on the next one (Figure 10). Details are in [22].

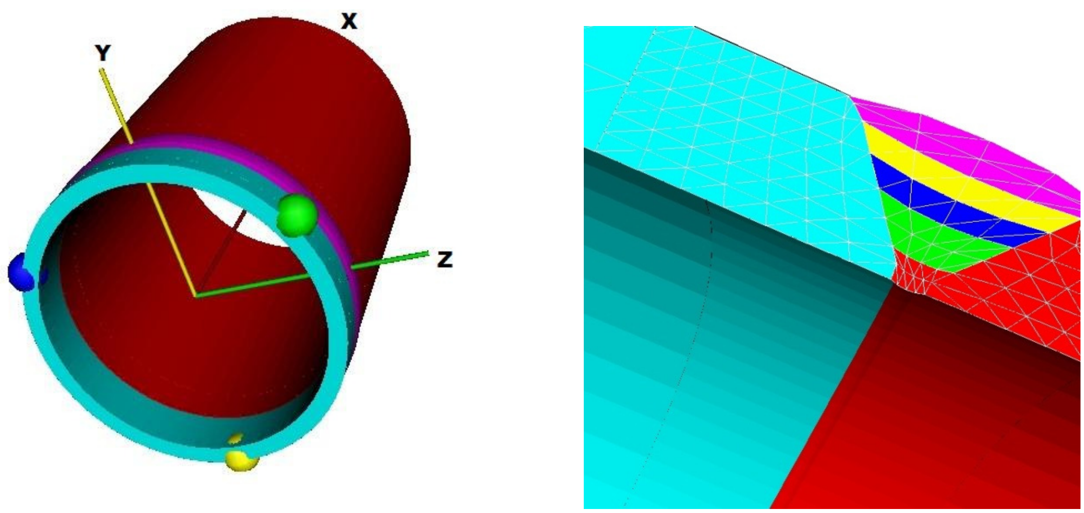

Figure 10 Welding sequence optimization setup

Conclusion

A total of 505 CWM analyses are implemented with 33 DOE matrices. Several examples of integrating computational optimization with CWM have been described. This would not be feasible using manual preparation and implementation. It is feasible by using an automated implementation with DOE matrices of a CWM problem without accumulated people-time to make multiple setups. It is argued that some parameters are more difficult to vary, e.g., varying the mesh geometry or the topology of the geometry and changing boundary condition types. However, for many types of parameters, the authors have shown that it is easy to set up parametric problems for CWM. The results presented here have demonstrated that using a DOE matrix in CWM is now practical for optimizing many decisions in the design of welded industrial structures. This is a powerful tool for a designer-driven optimization that enables a design group to do the optimization in the early design stages with knowledge of downstream welding and production engineering. This is in sharp contrast with the traditional practice of the designer waiting for the feedback from welding and production engineers to complete the design and handing off the optimization to a specialist. The authors argue that designer-driven optimization of the design of welded structures is now feasible for routine engineering in industry.

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